A New Hybrid Conjugates Gradient Algorithm For Unconstraint Optimization Problems

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Abstract

In this paper, we present a new hybrid conjugating gradient strategy that is both efficient and effective for solving unconstrained optimization problems. The parameter θ_k is derived from a convex combination of the β_k^{BA1} and the β_k^{FR} conjugating gradient methods. We demonstrated that this strategy is globally convergent under strong Wolfe line search conditions, and that the recommended hybrid CG method can create a descending search direction at each iteration. Numerical results are presented in this study, demonstrating that the proposed technique is both efficient and promising.

Keywords: Unconstrained Optimization, Conjugating gradient method, the descent property, Global convergence, Hybrid conjugating gradient method, Swc.

Introduction:

Let's assume we've got a function $f: R^n \rightarrow R$ which is continuously differentiable. Now let's consider the following unconstrained optimization problem

$$\text{Min}\{f(x): x \in R^n\} \quad (1)$$

Where R^n denotes an n-dimensional Euclidean space.

In order to solve Eq (1), we should start with an initial guess $x_0 \in R^n$, then we use a nonlinear conjugating gradient method to generate a sequence $\{x_k\}$ such as

$$x_{k+1} = x_k + \alpha_k d_k \quad (2)$$

where $\alpha_k > 0$ is achieved by line search and the direction d_k are generated as

$$d(x) = \begin{cases} -g_k, & k = 0 \\ -g_k + \beta_k d_{k-1}, & k > 0 \end{cases} \quad (3)$$

where $g_k = \nabla f(x)$ and β_k is a scalar parameter, which characterizes conjugating gradient methods.

Computing for the step-size α_k is said to satisfy any of the line search condition. In this paper we use the strong Wolfe line search.

$$f(x_k + \alpha_k d_k) \leq f(x) + \delta \alpha_k g_k^T d_k, \quad 0 \leq \delta \leq \frac{1}{2}$$

$$\left| d_k^T g(x_k + \alpha_k d_k) \right| \leq -\sigma g_k^T d_k, \quad \delta \leq \sigma \leq 1 \quad \textbf{(4)}$$

where d_k is the search direction which is clearly defined in Eq (3) .

For many years, researchers focused on the **CG** techniques. The outcome of those studies is several formulae with differences in **CG** coefficient (β_k) to solve unconstrained optimization problems.

Some common formula for β_k can be defined as:

$$\beta_k^{FR} = \frac{g_{k+1}^T g_{k+1}}{g_k^T g_k}$$, **FR (Fletcher-Reeves)** [1]

$$\beta_k^{PR} = \frac{g_{k+1}^T y_k}{g_k^T g_k}$$, **PR (Polak-Ribiere)** [2]

$$\beta_k^{DY} = \frac{g_k^T g_k}{d_{k-1}^T y_{k-1}}$$, **DY (Dai-Yuan)** [3]

$$\beta_k^{CD} = \frac{-g_k^T g_k}{d_{k-1}^T g_{k-1}}$$, **CD (conjugate descent)** [4]

$$\beta_k^{LS} = \frac{-g_k^T y_{k-1}}{d_{k-1}^T g_{k-1}}$$, **LS (Liu-Storey)** [5]

$$\beta_k^{HS} = \frac{g_k^T y_{k-1}}{d_{k-1}^T y_{k-1}}$$, **HS (Hestenes-Stiefel)** [6]

where $y_{k-1} = g_k - g_{k-1}$, and $\|.\|$ means the Euclidean norm. (5)

As we known that the **CG** methods β_k^{FR}, β_k^{CD} and β_k^{DY} have strongly global convergence properties, however, they have less computational performance. On the other hand, even though the β_k^{PR}, β_k^{LS} and β_k^{HS} methods haven‘t shown convergent all the time, however, they often give better computational performance.

In most cases, hybrid conjugating gradient methods are more efficient than basic conjugating gradient methods.

The hybrid conjugating gradient techniques discussed in this study are of particular importance. These algorithms are a mixture of different conjugating gradient techniques.

The primary concept behind their strategy is to make advantage of projections. They are commonly advocated as a way to avoid jamming. We proposed a new hybrid **CG** method which depends on BA_1 and FR methods, where the parameter $\beta_k^{BA_1}$ *and* β_k^{FR} are

$$\beta_k^{FR} = \frac{g_{k+1}^T g_{k+1}}{g_k^T g_k} \;,\; \beta_k^{BA_1} = \frac{y_k^T y_k}{-d_k^T g_k}$$ [7]

to solve the unconstrained optimization problems with suitable conditions.

The parameter $\beta_k{}^{H}$ in our proposed method is computed as a convex combination of β_k^{FR} and $\beta_k^{BA_1}$ such that

$$\beta_k^{HMB} = (1-\theta_k)\beta_k^{BA_1} + \theta_k \beta_k^{FR}$$
(6)

The remainder of the paper is formatted as follows: We present our proposed strategy for acquiring the parameter θ_k utilizing several methods in section 2. The sufficient descent property of our approach is also tested under certain conditions. Section 3 comprises numerous assumptions, whereas section 4 establishes the global convergence of the proposed approach. Section 5 summarizes the results of the numerical experiments that were conducted.

2_THE NEW HYBRID CONJUGATING GRADIENT METHOD

2.1 Derivation of the new parameter $\boldsymbol{\theta_k}$:The recurrence is used to calculate the iterates $x_0, x_1, x_2, \dots \dots$ of our algorithm (2). The step size $\alpha_k > 0$ is determined by the strong Wolfe conditions (4), and the directions are generated by the rule [8]

$$\begin{Bmatrix} d_0 = -g_0 \\ d_{k+1} = -g_{k+1} + \beta_k^{HMB} d_k \end{Bmatrix} \quad \textbf{(7)A}$$

Where $0 \le \theta_k \le 1$

$$\beta_k^{HMB} = (1-\theta_k)\beta_k^{BA_1} + \theta_k \beta_k^{FR} \quad \textbf{(7)B}$$

$$\beta_k^{HMB} = (1-\theta_k)\frac{y_k^T y_k}{-d_k^T g_k} + \theta_k \frac{g_{k+1}^T g_{k+1}}{g_k^T g_k}$$

$$d_{k+1} = -g_{k+1} + (1-\theta_k)\frac{y_k^T y_k}{-d_k^T g_k} d_k + \theta_k \frac{g_{k+1}^T g_{k+1}}{g_k^T g_k} d_k$$

$$y_k^T d_{k+1} = -y_k^T g_{k+1} + (1-\theta_k)\frac{y_k^T y_k}{-d_k^T g_k} y_k^T d_k + \theta_k \frac{g_{k+1}^T g_{k+1}}{g_k^T g_k} y_k^T d_k$$

Hence, from the conjugacy condition $y_k^T d_{k+1} = 0$ [9]

we get $0 = -y_k^T g_{k+1} + (1-\theta_k)\frac{y_k^T y_k}{-d_k^T g_k} y_k^T d_k + \theta_k \frac{g_{k+1}^T g_{k+1}}{g_k^T g_k} y_k^T d_k$

$$\theta_k = \frac{\left[\frac{y_k^T g_{k+1}}{y_k^T d_k} + \frac{y_k^T y_k}{d_k^T g_k}\right] y_k^T d_k}{\left[\frac{y_k^T y_k}{d_k^T g_k} + \frac{g_{k+1}^T g_{k+1}}{g_k^T g_k}\right] y_k^T d_k}$$

or

$$\theta_k = \frac{\left[\frac{y_k^T g_{k+1}}{y_k^T d_k} + \frac{y_k^T y_k}{d_k^T g_k}\right]}{\left[\frac{y_k^T y_k}{d_k^T g_k} + \frac{g_{k+1}^T g_{k+1}}{g_k^T g_k}\right]} \quad \textbf{(8)}$$

We see that when $\theta_k = 0$ then $\beta_k^{HMB} = \beta_k^{BA_1}$ and when $\theta_k = 1$ then β_k^{HMB} reduced to the second part β_k^{FR}. On the other hand, if $0 < \theta_k < 1$, then β_k^{HMB} is a convex combination of $\beta_k^{BA_1}$ and β_k^{FR}

2.2 The New Algorithm

Step1: initialization select $x_0 \in R^n$ and the parameters $0 < \delta < \sigma < 1$, compute $f(x_0)$ and g_0. Consider

$d_0 = -g_0$ and set $\alpha_0 = \frac{1}{\|g_0\|}$ when n = 0

Step2: The stopping criterion i.e. $\|g_k\| \leq 10^{-6}$ then stop.

Step3: line search compute $\alpha_k = \alpha_{k-1}\frac{\|d_{k-1}\|}{\|d_k\|}$, the step size must $\alpha_k > 0$ and satisfy the strong Wolfe line search condition (4) .

Step4: Calculate θ_k as in (8) with $0 < \theta_k < 1$, then compute β_k^H

conjugate gradient parameter as in (7)B.

Step5:Generate $d_{k+1} = -g_{k+1} + \beta_k^H d_k$, and update the variables $x_{k+1} = x_k + \alpha_k d_k$.

Compute $f(x_{k+1})$, g_{k+1} and $s_k = x_{k+1} - x_k$, $y_k = g_{k+1} - g_k$.

Step6: If the restart criteria of Powell $|g_{k+1}^T g_k| \geq 0.2\|g_{k+1}\|^2$ is satisfied, then set $d_k = -g_{k+1}$

Otherwise put $d_{k+1} = d_k$

Step7: set $k = k + 1$ and continue with step2.

3_ THE DESCENT PROPERTY

Hypothesis H

$H1$: The objective function $f(x)$ is a continuously differentiable function, which means it can be decomposed into two parts.

The level set L_1 ={$x \in R^n$: $f(x) \leq f(x_1)$} at $x_1$is bounded (x_1 $is\ the\ initail\ point$), namely, there exists a constant $a > 0$ such that

$$\|x\| \leq a \ \ for\ all\ x \in L_1$$

H_2**:** In every neighborhood N of L1, f is continuously differentiable, and its gradient $g(x)$ is Lipschitz continuous with Lipschitz constant L>0, i.e., f is continuously differentiable in any neighborhood N of L_1.

$\|\nabla f(x) - \nabla f(y)\| \leq L\ \|x - y\| for\ all\ x, y\ \in N$ [10]

Lemma 1.

Let's assume that the goal function meets the requirements of assumption H. Take an example procedure (2). (3) The following is true when α_k is satisfied by the strong Wolfe line search (4) and β_k^H is satisfying the formula (6).

$g_{k+1}^T d_{k+1} < 0$ for all k

Proof:

For $k = 1$ we have $g_1^T d_1 = -g_1^T g_1 = -\|g_1\|^2 < 0$ according to $d_1 = -g_1$

For $k > 1$, suppose that $g_k^T d_k < 0$, holds at the $k - th$ step i.e.: $g_k^T d_k = -c\|g_1\|^2 < 0$, then we prove this inequality also holds at the $(k+1) - th$ step. Multiply (7)a by g_{k+1}^T we get $|g_{k+1}^T d_k|$

$$g_{k+1}^T d_{k+1} = -g_{k+1}^T g_{k+1} + (1-\theta_k)\frac{y_k^T y_k}{-d_k^T g_k}\ g_{k+1}^T\, d_k + \theta_k \frac{g_{k+1}^T g_{k+1}}{g_k^T g_k}\ g_{k+1}^T d_k$$

$$d_{k+1}^T g_{k+1} = -g_{k+1}^T g_{k+1} + \frac{y_k^T y_k}{-d_k^T g_k}\ d_k^T\, g_{k+1} - \theta_k \frac{y_k^T y_k}{-d_k^T g_k}\ d_k^T\, g_{k+1} + \theta_k \frac{g_{k+1}^T g_{k+1}}{g_k^T g_k}\ d_k^T\, g_{k+1}$$

$$= -g_{k+1}^T g_{k+1} + \frac{(g_{k+1}-g_k)^T(g_{k+1}-g_k)}{-d_k^T g_k} d_k^T\, g_{k+1} - \theta_k \frac{(g_{k+1}-g_k)^T(g_{k+1}-g_k)}{-d_k^T g_k} d_k^T\, g_{k+1} + \theta_k \frac{g_{k+1}^T g_{k+1}}{g_k^T g_k}\ d_k^T\, g_{k+1}$$

$$= -\|g_{k+1}\|^2 + \frac{\|g_{k+1}\|^2}{-d_k^T g_k}\ d_k^T g_{k+1} - 2\ \frac{g_{k+1}^T g_k}{-d_k^T g_k}\ d_k^T g_{k+1} + \frac{\|g_k\|^2}{-d_k^T g_k}\ d_k^T g_{k+1} - \theta_k\ \frac{\|g_{k+1}\|^2}{-d_k^T g_k}\ d_k^T g_{k+1} + 2\,\theta_k \frac{g_{k+1}^T g_k}{-d_k^T g_k}\ d_k^T g_{k+1} - \theta_k\ \frac{\|g_k\|^2}{-d_k^T g_k}\ d_k^T g_{k+1} + \theta_k\ \frac{\|g_{k+1}\|^2}{g_k^T g_k}\ d_k^T g_{k+1}$$

$$= -\|g_{k+1}\|^2 - \frac{\|g_{k+1}\|^2}{d_k^T g_k}\ d_k^T g_{k+1} + 2\ \frac{g_{k+1}^T g_k}{d_k^T g_k}\ d_k^T g_{k+1} - \frac{\|g_k\|^2}{d_k^T g_k}\ d_k^T g_{k+1} + \theta_k\ \frac{\|g_{k+1}\|^2}{d_k^T g_k}\ d_k^T g_{k+1} - 2\,\theta_k \frac{g_{k+1}^T g_k}{d_k^T g_k}\ d_k^T g_{k+1} + \theta_k\ \frac{\|g_k\|^2}{d_k^T g_k}\ d_k^T g_{k+1} + \theta_k\ \frac{\|g_{k+1}\|^2}{g_k^T g_k}\ d_k^T g_{k+1}$$

$$\boldsymbol{\sigma g_k^T d_k \le g_{k+1}^T d_k \le -\sigma\, g_k^T d_k}$$

$$\boldsymbol{g_{k+1}^T g_k \le -\psi \|g_{k+1}\|^2} \quad [11]$$

$$d_{k+1}^T g_{k+1} \le -\|g_{k+1}\|^2 + \sigma \frac{\|g_{k+1}\|^2}{d_k^T g_k}\ d_k^T g_k + 2\sigma\psi\ \frac{\|g_{k+1}\|^2}{d_k^T g_k}\ d_k^T g_k + \sigma \frac{\|g_k\|^2}{d_k^T g_k}\ d_k^T g_k - \theta_k \sigma\ \frac{\|g_{k+1}\|^2}{d_k^T g_k}\ d_k^T g_k - 2\sigma\psi\,\theta_k \frac{\|g_{k+1}\|^2}{d_k^T g_k}\ d_k^T g_k - \theta_k \sigma \frac{\|g_k\|^2}{d_k^T g_k}\ d_k^T g_k - \theta_k\, \sigma\ \frac{\|g_{k+1}\|^2}{g_k^T g_k}\ d_k^T g_k$$

$$\boldsymbol{g_k^T d_k} \le -\boldsymbol{c}\|\boldsymbol{g_k}\|^2$$

$$d_{k+1}^T g_{k+1} \le -\|g_{k+1}\|^2 + \sigma\|g_{k+1}\|^2 + 2\sigma\psi\|g_{k+1}\|^2 + \sigma\frac{\|g_k\|^2}{\|g_{k+1}\|^2}\|g_{k+1}\|^2 - \theta_k\sigma\|g_{k+1}\|^2 - 2\sigma\psi\,\theta_k\|g_{k+1}\|^2 - \theta_k\sigma\frac{\|g_k\|^2}{\|g_{k+1}\|^2}\|g_{k+1}\|^2 + c\sigma\theta_k\|g_{k+1}\|^2$$

$$d_{k+1}^T g_{k+1} \le -\left[1 - \sigma - 2\sigma\psi - \frac{\sigma}{\beta_k^{FR}} + \sigma\,\theta_k + 2\sigma\psi\,\theta_k + \frac{\sigma\theta_k}{\beta_k^{FR}} - c\sigma\theta_k\right]\|g_{k+1}\|^2$$

$$d_{k+1}^T g_{k+1} \le -C_1\|g_{k+1}\|^2 \qquad 0 < C_1 < 1$$

$$C_1 = \left[1 - \sigma - 2\sigma\psi - \frac{\sigma}{\beta_k^{FR}} + \sigma\,\theta_k + 2\sigma\psi\,\theta_k + \frac{\sigma\theta_k}{\beta_k^{FR}} - c\sigma\theta_k\right]$$

4-Global convergence.

Theorem 4.1.

Let's suppose that the assumption H_1 and H_2 holds. Consider the algorithm (2),(7),(8) where $0 \le \theta_k \le 1$ and $\alpha_k > 0$ is obtained by the strong Wolfe line search.

If $\|s_k\|$ tends to zero and there exists non-negative constant η_1 and η_2 such that $\|g_k\|^2 \ge \eta_1\|s_k\|^2$; $\|g_{k+1}\|^2 \le \eta_2\|s_k\|$

and f is uniformly convex function, then $\lim_{k\to\infty} g_k = 0$

Lemma 4.1:

If the assumptions H_1 and H_2 are true, we may examine any conjugating gradient (2) or (3), where d_k is the descent direction and $\alpha_k > 0$ is the result of a strong Wolfe line searching to determine the gradient. If

$$\sum_{k\ge1}\frac{1}{\|d_{k+1}\|^2} < \infty \qquad \text{then}$$

$$\liminf_{K\to\infty}\|g_k\| = 0 \;\; [12]$$

Proof:

$$\beta_k^{HMB} = (1-\theta_k)\beta_k^{BA_1} + \theta_k\beta_k^{FR}$$

$$\beta_k^{HMB} = (1-\theta_k)\frac{y_k^T y_k}{-d_k^T g_k} + \theta_k\frac{g_{k+1}^T g_{k+1}}{g_k^T g_k}$$

$$\beta_k^{HMB} \leq \frac{\|y_k\|^2}{-d_k^T g_k} + \frac{\|g_{k+1}\|^2}{\|g_k\|^2}$$

From $\|y_k\| \leq \mathrm{L}\|S_k\|$

$$\leq \frac{L^2\|S_k\|^2}{c\|g_k\|^2} + \frac{\|g_{k+1}\|^2}{\|g_k\|^2}$$

$$\beta_k^{HMB} \leq \frac{L^2\|S_k\|^2}{c\eta_1\|S_k\|^2} + \frac{\eta_2\,\|S_k\|}{\eta_1\,\|S_k\|^2}$$

$$\beta_k^{HMB} \leq \frac{L^2\|S_k\|}{c\eta_1\|S_k\|} + \frac{\eta_2}{\eta_1\,\|S_k\|}$$

The new direction

$$d_{k+1} = -g_{k+1} + \beta_k^{HMB}\, d_k$$

$$\|d_{k+1}\| = \|-g_{k+1} + \beta_k^{HMB}\, d_k\| \leq \|g_{k+1}\| + |\beta_k^{HMB}|\|d_k\|$$

$$\|d_{k+1}\|^2 = \|g_{k+1}\|^2 + 2\beta_k^{HMB}\|g_{k+1}\|\|d_k\| + (\beta_k^{HMB})^2\|d_k\|^2$$

$$\leq \eta_2\,\|S_k\| + 2\left[\frac{L^2\|S_k\|}{c\eta_1\|S_k\|} + \frac{\eta_2}{\eta_1\,\|S_k\|}\right]\eta_2^{\frac{1}{2}}\|S_k\|^{\frac{1}{2}}\frac{\|S_k\|}{|\alpha_k|}$$

$$+ \left[\frac{L^2\|S_k\|}{c\eta_1\|S_k\|} + \frac{\eta_2}{\eta_1\,\|S_k\|}\right]^2\frac{\|S_k\|^2}{|\alpha_k|^2}$$

From $\|S_k\| \leq D$

$$\leq \eta_2\,D + 2\left[\frac{L^2 D}{c\eta_1} + \frac{\eta_2}{\eta_1}\right]\eta_2^{\frac{1}{2}}\frac{D^{\frac{1}{2}}}{|\alpha_k|} + \left[\frac{L^2 D}{c\eta_1} + \frac{\eta_2}{\eta_1}\right]^2\frac{1}{|\alpha_k|^2}$$

let $\varphi = \eta_2\,D + 2\left[\frac{L^2 D}{c\eta_1} + \frac{\eta_2}{\eta_1}\right]\eta_2^{\frac{1}{2}}\frac{D^{\frac{1}{2}}}{|\alpha_k|} + \left[\frac{L^2 D}{c\eta_1} + \frac{\eta_2}{\eta_1}\right]^2\frac{1}{|\alpha_k|^2}$

$\therefore \|d_{k+1}\|^2 \leq \varphi$

$$\sum_{k\geq 1}\frac{1}{\|d_{k+1}\|^2} \geq \sum_{k\geq 1}\frac{1}{\varphi} = \frac{1}{\varphi}\sum 1 = \frac{1}{\varphi}\,\infty = \infty$$

$$\therefore \lim_{k\to\infty} inf\,\|g_k\| = 0$$

5-Numerical

In this section, we'll discuss the results of our numerical experiments with the hybrid MB algorithm and compare them to the numerical results of the other two algorithms (FR, BA1) under the strong Wolfe line search, which is based on number of iterations (NI) and number of function evaluation (NF),

with iterations ending when $\|g_k\| \leq 10^{-6}$.

In addition, when the number of variables (n=200,900) was high, we used 75 functions of unconstrained optimization problems. All the graphs in this study were created in Fortran.

The results are shown in Table 1.

TABLE 1. list numerical result details.

Function	The dimension	HMB NI	HMB NF	FR NI	FR NF	BA1 NI	BA1 NF
Extended Trigonometric	200	22	39	23	39	33	58
	900	31	57	36	60	48	83
Extended Rosenbrock	200	35	78	38	80	75	143
	900	35	78	40	86	1001	1513
Extended White &Holst	200	36	80	40	85	53	103
	900	29	57	39	82	1001	1539
Extended Beale	200	14	27	16	30	34	68
	900	14	27	15	28	30	64
Raydan 1	200	123	191	1001	1075	647	995
	900	403	689	468	817	1001	1560
Raydan 2	200	4	9	4	9	4	9
	900	4	9	4	9	4	9
Diagonal 1 2	200	96	158	99	166	434	678
	900	197	320	209	351	1001	1515

		HMB		FR		BA1	
Function	The dimension	NI	NF	NI	NF	NI	NF
Generalized Tridiagonal 2	200	51	76	52	76	76	129
	900	58	93	64	104	93	160
Extended Himmelblau	200	10	19	10	19	42	81
	900	11	21	22	35	28	50
Extended psc1	200	7	15	7	15	26	151
	900	7	15	7	15	11	27
Extended powell	200	70	128	80	147	1001	1503
	900	80	150	90	169	1001	1531
Extended Maratos	200	69	164	70	149	1001	1120
	900	76	180	101	402	166	572
Extended Wood	200	24	47	25	49	236	454
	900	25	49	28	54	1001	1522
Extended Hiepert	200	79	174	90	195	106	231
	900	79	171	86	184	114	244
Extended Quadratic penalty Qp1	200	23	427	100	3016	69	2007
	900	8	21	8	21	45	626
Quadratic Qf 2	200	157	250	163	256	747	1142
	900	368	573	1001	1203	1001	1501
Extended Tridiagonal 2	200	35	54	35	53	55	95
	900	47	69	61	635	57	105
ARWHEAD	200	8	15	8	15	1001	1030
	900	14	85	20	247	69	804

		HMB		FR		BA1	
Function	The dimension	NI	NF	NI	NF	NI	NF
NONDIA	200	11	21	15	30	27	55
	900	13	26	17	33	18	38
DIXMAANA	200	7	14	7	14	7	14
	900	7	14	7	14	13	27
DIXMAANC	200	13	23	13	23	17	32
	900	14	25	14	25	16	64
Tridiagonal perturbed Quadratic	200	127	203	161	254	1001	1515
	900	285	450	338	515	1001	1513
EDENSCH	200	25	46	25	48	1001	1028
	900	38	385	85	1738	1001	1037
LIARWHD	200	16	36	19	40	1001	1509
	900	19	44	21	45	1001	1513
ENGVAL1	200	77	1506	73	1581	205	5420
	900	26	281	147	4029	147	3430
Extended DENCHNA	200	9	16	11	19	25	46
	900	19	31	22	36	26	50
Extended DENCHNB	200	7	15	7	15	20	39
	900	7	15	7	15	10	21
Extended Block-Diagonal	200	11	20	12	23	41	70
	900	10	19	11	21	40	69
	200	7	18	7	18	20	42

Function	The dimension	HMB		FR		BA1	
		NI	NF	NI	NF	NI	NF
Generalized quartic GQ1	900	7	18	7	18	10	24
SINCOS	200	7	15	7	15	26	151
	900	7	15	7	15	11	27
FLETCHCR	200	21	45	22	47	37	67
	900	27	54	28	54	47	82
Extended Himmelblau	200	6	13	6	13	19	37
	900	6	13	6	13	15	29

The percentage of improvement is shown in both tables 2-3

Table 2

Measures	$\boldsymbol{\beta_k^{HMB}}$	$\boldsymbol{\beta_k^{FR}}$
NI200	47%	**100**%
NF200	49%	**100**%
NI900	35%	**100**%
NF900	64%	**100**%

Table 3:

Measures	$\boldsymbol{\beta_k^{HMB}}$	$\boldsymbol{\beta_k^{BA1}}$
NI200	86%	**100**%
NF200	81%	**100**%
NI900	83%	**100**%
NF900	81%	**100**%

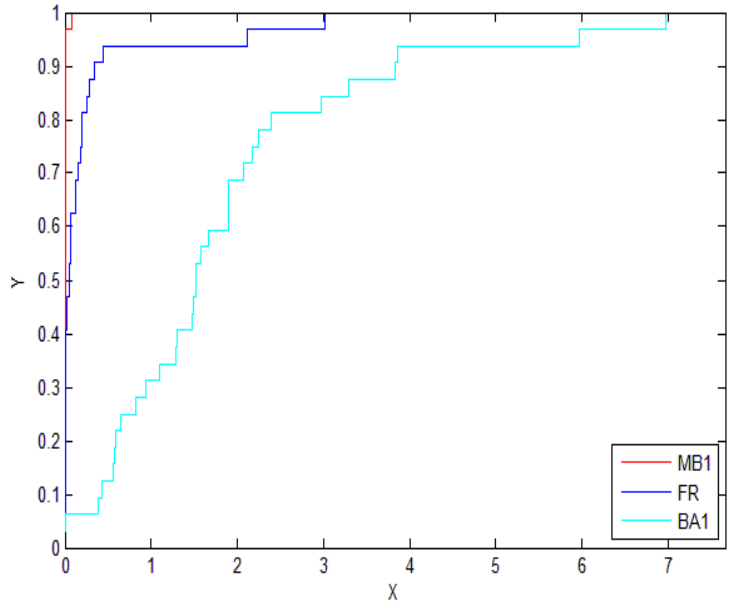

Figure 1:Based on NI at 200

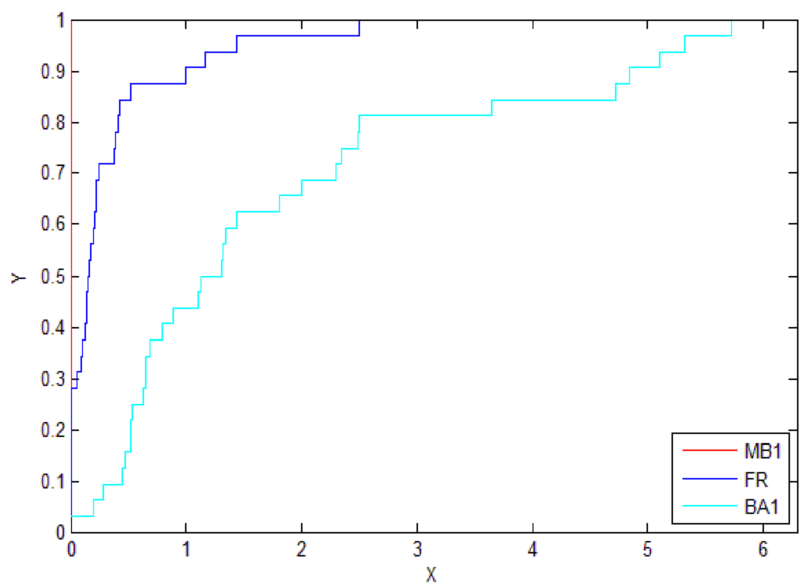

Figure 2: Based on NI 900

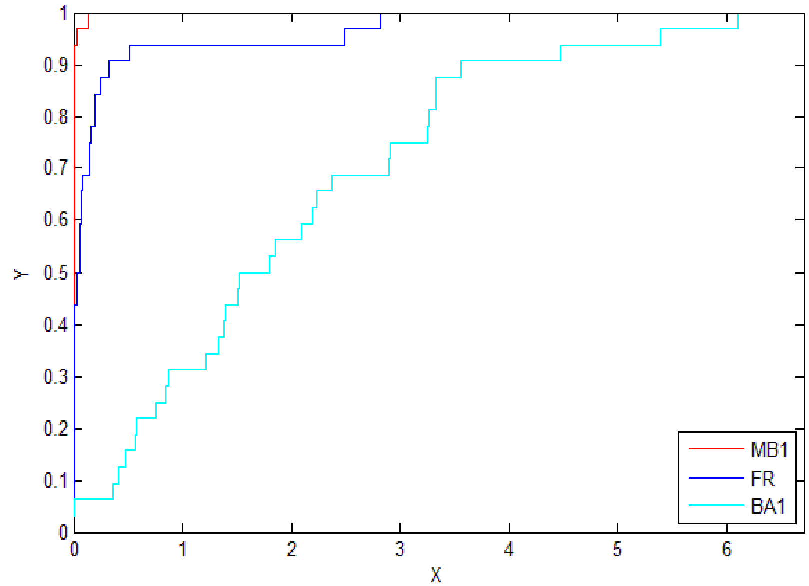

Figure 3: Based on NF 200

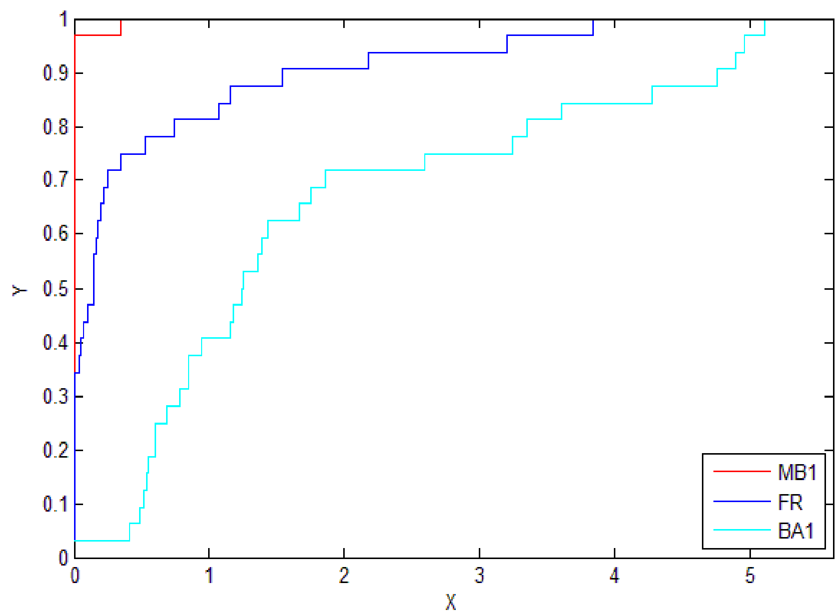

Figure 4: Based on NF 900

The statistics above show a comparison of the new algorithm MB with both FR and BA1 in terms of NI, NF. Dolan and More [13] is utilized to demonstrate the outcomes of a newly developed hybrid conjugate gradient algorithm. As a result, we can deduce that the hybrid method is effective.

6-Conclusion:

Based on the hybridization of the two algorithms ($\beta_k^{BA_1}$ and β_k^{FR}), a new approach termed was introduced in this research for hybrid conjugating gradient in unconstrained optimization.

The qualities of sufficient descent and global convergence of the suggested algorithm have been confirmed by some of the assumptions employed, and the proposed method has been explored both theoretically and practically.

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