A New Hybrid Conjugates Gradient Algorithm For Unconstrain Optimization Problems

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Abstract

In this paper, we present a new hybrid conjugating gradient strategy that is both efficient and effective for solving unconstrained optimization problems. The parameter \( \theta_k \) is derived from a convex combination of the \( \beta_k^{BA} \) and the \( \beta_k^{FR} \) conjugating gradient methods. We demonstrated that this strategy is globally convergent under strong Wolfe line search conditions, and that the recommended hybrid CG method can create a descending search direction at each iteration. Numerical results are presented in this study, demonstrating that the proposed technique is both efficient and promising.

Keywords: Unconstrained Optimization, Conjugating gradient method, the descent property, Global convergence, Hybrid conjugating gradient method, Swc.

Introduction:

Let’s assume we’ve got a function \( f: R^n \rightarrow R \) which is continuously differentiable. Now let’s consider the following unconstrained optimization problem

\[
\text{Min}\{f(x); x \in R^n\} \tag{1}
\]

Where \( R^n \) denotes an \( n \)-dimensional Euclidean space.

In order to solve Eq (1), we should start with an initial guess \( x_0 \in R^n \), then we use a nonlinear conjugating gradient method to generate a sequence \( \{x_k\} \) such as

\[
x_{k+1} = x_k + \alpha_k d_k \tag{2}
\]

where \( \alpha_k > 0 \) is achieved by line search and the direction \( d_k \) are generated as

\[
d(x) = \begin{cases} 
-g_k, & k = 0 \\
g_k + \beta_k d_{k-1}, & k > 0 
\end{cases} \tag{3}
\]

where \( g_k = \nabla f(x) \) and \( \beta_k \) is a scalar parameter, which characterizes conjugating gradient methods.

Computing for the step-size \( \alpha_k \) is said to satisfy any of the line search condition. In this paper we use the strong Wolfe line search.

\[
f(x_k + \alpha_k d_k) \leq f(x) + \delta \alpha_k g_k^T d_k, \quad 0 \leq \delta \leq \frac{1}{2}
\]

\[
|d_k^T g(x_k + \alpha_k d_k)| \leq -\sigma g_k^T d_k, \quad \delta \leq \sigma \leq 1 \tag{4}
\]
where $d_k$ is the search direction which is clearly defined in Eq (3).

For many years, researchers focused on the CG techniques. The outcome of those studies is several formulae with differences in CG coefficient ($\beta_k$) to solve unconstrained optimization problems.

Some common formula for $\beta_k$ can be defined as:

$$
\beta_k^{FR} = \frac{\theta_k^T \theta_{k+1}}{\theta_k^T \theta_k}, \text{ FR (Fletcher-Reeves) [1]}
$$

$$
\beta_k^{PR} = \frac{\theta_k^T \gamma_k}{\theta_k^T \theta_k}, \text{ PR (Polak-Ribiere) [2]}
$$

$$
\beta_k^{DY} = \frac{\theta_k^T \theta_k - 1}{d_{k-1}^T \gamma_{k-1}}, \text{ DY (Dai-Yuan) [3]}
$$

$$
\beta_k^{CD} = \frac{-\theta_k^T \theta_k}{d_{k-1}^T \gamma_{k-1}}, \text{ CD (conjugate descent) [4]}
$$

$$
\beta_k^{LS} = \frac{-\theta_k^T \gamma_{k-1}}{d_{k-1}^T \gamma_{k-1}}, \text{ LS (Liu-Storey) [5]}
$$

$$
\beta_k^{HS} = \frac{-\theta_k^T \gamma_{k-1}}{d_{k-1}^T \gamma_{k-1}^{\frac{1}{2}}}, \text{ HS (Hestenes-Stiefel) [6]}
$$

where $\gamma_{k-1} = g_k - g_{k-1}$, and $\| \|$ means the Euclidean norm. (5)

As we known that the CG methods $\beta_k^{FR}$, $\beta_k^{CD}$ and $\beta_k^{DY}$ have strongly global convergence properties, however, they have less computational performance. On the other hand, even though the $\beta_k^{PR}$, $\beta_k^{LS}$ and $\beta_k^{HS}$ methods haven’t shown convergent all the time, however, they often give better computational performance.

In most cases, hybrid conjugating gradient methods are more efficient than basic conjugating gradient methods.

The hybrid conjugating gradient techniques discussed in this study are of particular importance. These algorithms are a mixture of different conjugating gradient techniques.

The primary concept behind their strategy is to make advantage of projections. They are commonly advocated as a way to avoid jamming. We proposed a new hybrid CG method which depends on $BA_1$ and FR methods, where the parameter $\beta_k^{BA_1}$ and $\beta_k^{FR}$ are

$$
\beta_k^{FR} = \frac{\theta_k^T \theta_{k+1}}{\theta_k^T \theta_k}, \beta_k^{BA_1} = \frac{\gamma_k^T \gamma_k}{d_k^T \theta_k} \text{ [7]}
$$

to solve the unconstrained optimization problems with suitable conditions.

The parameter $\beta_k^H$ in our proposed method is computed as a convex combination of $\beta_k^{FR}$ and $\beta_k^{BA_1}$ such that

$$
\beta_k^{HMB} = (1 - \theta_k) \beta_k^{BA_1} + \theta_k \beta_k^{FR} \text{ (6)}
$$

The remainder of the paper is formatted as follows: We present our proposed strategy for acquiring the parameter $\theta_k$ utilizing several methods in section 2. The sufficient descent property of our approach is also tested under certain
conditions. Section 3 comprises numerous assumptions, whereas section 4 establishes the global convergence of the proposed approach. Section 5 summarizes the results of the numerical experiments that were conducted.

2. THE NEW HYBRID CONJUGATING GRADIENT METHOD

2.1 Derivation of the new parameter $\theta_k$: The recurrence is used to calculate the iterates $x_0, x_1, x_2, \ldots$ of our algorithm (2). The step size $\alpha_k > 0$ is determined by the strong Wolfe conditions (4), and the directions are generated by the rule [8]

$$
\begin{cases}
    d_0 = -g_0 \\
    d_{k+1} = -g_{k+1} + \beta_k^{HMB} d_k
\end{cases}
$$

Where $0 \leq \theta_k \leq 1$

$$
\beta_k^{HMB} = (1 - \theta_k) \beta_k^{BA} + \theta_k \beta_k^{FR}
$$

$$
d_{k+1} = -g_{k+1} + (1 - \theta_k) \frac{y_k^T y_k}{d_k^T g_k} d_k + \theta_k \frac{g_k^T g_{k+1}}{g_k^T g_k} d_k
$$

$$
y_k^T d_{k+1} = -y_k^T g_{k+1} + (1 - \theta_k) \frac{y_k^T y_k}{d_k^T g_k} y_k^T d_k + \theta_k \frac{g_k^T g_{k+1}}{g_k^T g_k} y_k^T d_k
$$

Hence, from the conjugacy condition $y_k^T d_{k+1} = 0$ [9]

we get

$$
0 = -y_k^T g_{k+1} + (1 - \theta_k) \frac{y_k^T y_k}{d_k^T g_k} y_k^T d_k + \theta_k \frac{g_k^T g_{k+1}}{g_k^T g_k} y_k^T d_k
$$

$$
\theta_k = \left[ \frac{y_k^T g_{k+1} + \frac{y_k^T y_k}{d_k^T g_k} y_k^T d_k}{y_k^T g_k + \frac{g_k^T g_{k+1}}{g_k^T g_k} d_k^T g_k} \right] y_k^T d_k
$$

or

$$
\theta_k = \frac{y_k^T g_{k+1} + \frac{y_k^T y_k}{d_k^T g_k} d_k^T g_k}{y_k^T g_k + \frac{g_k^T g_{k+1}}{g_k^T g_k} d_k^T g_k}
$$

(8)
We see that when \( \theta_k = 0 \) then \( \beta_k^{HMB} = \beta_k^{BA} \) and when \( \theta_k = 1 \) then \( \beta_k^{HMB} \) reduced to the second part \( \beta_k^{FR} \). On the other hand, if \( 0 < \theta_k < 1 \), then \( \beta_k^{HMB} \) is a convex combination of \( \beta_k^{BA} \) and \( \beta_k^{FR} \).

### 2.2 The New Algorithm

**Step1:** initialization select \( x_0 \in \mathbb{R}^n \) and the parameters \( 0 < \delta < \sigma < 1 \), compute \( f(x_0) \) and \( g_0 \). Consider \( d_0 = -g_0 \) and set \( \alpha_0 = \frac{1}{\|g_0\|} \) when \( n = 0 \).

**Step2:** The stopping criterion i.e. \( \|g_k\| \leq 10^{-6} \) then stop.

**Step3:** line search compute \( \alpha_k = \frac{\|d_{k-1}\|}{\|d_k\|} \), the step size must \( \alpha_k > 0 \) and satisfy the strong Wolfe line search condition (4).

**Step4:** Calculate \( \theta_k \) as in (8) with \( 0 < \theta_k < 1 \), then compute \( \beta_k^H \) conjugate gradient parameter as in (7B).

**Step5:** Generate \( d_{k+1} = -g_{k+1} + \beta_k^H d_k \), and update the variables \( x_{k+1} = x_k + \alpha_k d_k \).

Compute \( f(x_{k+1}), g_{k+1} \) and \( s_k = x_{k+1} - x_k, y_k = g_{k+1} - g_k \).

**Step6:** If the restart criteria of Powell \( |g_{k+1}^T g_k| \geq 0.2 \|g_{k+1}\|^2 \) is satisfied, then set \( d_k = -g_{k+1} \).

Otherwise put \( d_{k+1} = d_k \).

**Step7:** set \( k = k + 1 \) and continue with step2.

### 3. THE DESCENT PROPERTY

**Hypothesis H**

\( H1: \) The objective function \( f(x) \) is a continuously differentiable function, which means it can be decomposed into two parts.

The level set \( L_1 = \{ x \in \mathbb{R}^n : f(x) \leq f(x_1) \} \) at \( x_1 \) is bounded (\( x_1 \) is the initial point), namely, there exists a constant \( a > 0 \) such that

\[ \|x\| \leq a \text{ for all } x \in L_1 \]

\( H2: \) In every neighborhood \( N \) of \( L_1 \), \( f \) is continuously differentiable, and its gradient \( g(x) \) is Lipschitz continuous with Lipschitz constant \( L > 0 \), i.e., \( f \) is continuously differentiable in any neighborhood \( N \) of \( L_1 \).

\[ \|\nabla f(x) - \nabla f(y)\| \leq L \|x - y\| \text{ for all } x, y \in N \]

**Lemma 1.**

Let’s assume that the goal function meets the requirements of assumption H. Take an example procedure (2). (3) The following is true when \( \alpha_k \) is satisfied by the strong Wolfe line search (4) and \( \beta_k^H \) is satisfying the formula (6).
\[ g_{k+1}d_{k+1} < 0 \] for all \( k \)

**Proof:**

For \( k = 1 \) we have \( g_1^Td_1 = -g_1^Tg_1 = -\|g_1\|^2 < 0 \) according to \( d_1 = -g_1 \)

For \( k > 1 \), suppose that \( g_k^Td_k < 0 \), holds at the \( k - \)th step i.e.: \( g_k^Td_k = -c\|g_k\|^2 < 0 \), then we prove this inequality also holds at the \((k+1) - \)th step. Multiply (7) by \( g_{k+1}^T \) we get \( |g_k^Td_k| \)

\[
\begin{align*}
g_{k+1}d_{k+1} &= -g_{k+1}^Tg_{k+1} + (1 - \theta_k) \frac{y_k^Ty_k}{-d_k^Tg_k} g_{k+1}^T g_k + \theta_k \frac{g_{k+1}^T g_{k+1}}{g_k^T g_k} d_{k+1}^T g_k \\
d_{k+1} g_{k+1} &= -g_{k+1}^Tg_{k+1} + \frac{y_k^Ty_k}{-d_k^Tg_k} d_k^T g_{k+1} - \theta_k \frac{y_k^Ty_k}{-d_k^Tg_k} d_k^T g_{k+1} + \theta_k \frac{g_{k+1}^T g_{k+1}}{g_k^T g_k} d_k^T g_{k+1}
\end{align*}
\]

\[
\begin{align*}
&= -g_{k+1}^T g_{k+1} + \frac{(g_{k+1} - g_k)^T (g_{k+1} - g_k)}{-d_k^T g_k} d_k^T g_{k+1} - \theta_k \frac{(g_{k+1} - g_k)^T (g_{k+1} - g_k)}{-d_k^T g_k} d_k^T g_{k+1} \\
&\quad + \theta_k \frac{g_{k+1}^T g_{k+1}}{g_k^T g_k} d_k^T g_{k+1}
\end{align*}
\]

\[
\begin{align*}
&= -\|g_{k+1}\|^2 + \frac{\|g_{k+1}\|^2}{-d_k^T g_k} d_k^T g_{k+1} - 2 \frac{g_{k+1}^T g_k}{-d_k^T g_k} d_k^T g_{k+1} + \frac{\|g_k\|^2}{-d_k^T g_k} d_k^T g_{k+1} - \theta_k \frac{\|g_{k+1}\|^2}{-d_k^T g_k} d_k^T g_{k+1} \\
&\quad + 2 \theta_k \frac{g_{k+1}^T g_k}{-d_k^T g_k} d_k^T g_{k+1} - \theta_k \frac{\|g_k\|^2}{-d_k^T g_k} d_k^T g_{k+1} + \theta_k \frac{\|g_{k+1}\|^2}{g_k^T g_k} d_k^T g_{k+1}
\end{align*}
\]

\[
\begin{align*}
&= -\|g_{k+1}\|^2 - \frac{\|g_{k+1}\|^2}{d_k^T g_k} d_k^T g_{k+1} + 2 \frac{g_{k+1}^T g_k}{d_k^T g_k} d_k^T g_{k+1} - \frac{\|g_k\|^2}{d_k^T g_k} d_k^T g_{k+1} + \theta_k \frac{\|g_{k+1}\|^2}{d_k^T g_k} d_k^T g_{k+1} \\
&\quad - 2 \theta_k \frac{g_{k+1}^T g_k}{d_k^T g_k} d_k^T g_{k+1} + \theta_k \frac{\|g_k\|^2}{d_k^T g_k} d_k^T g_{k+1} + \theta_k \frac{\|g_{k+1}\|^2}{g_k^T g_k} d_k^T g_{k+1}
\end{align*}
\]

\[
\begin{align*}
\sigma g_k^Td_k &\leq g_{k+1}^Td_k \\
g_{k+1}^Td_k &\leq -\psi \|g_{k+1}\|^2 \quad [11]
\end{align*}
\]

\[
\begin{align*}
d_{k+1}^T g_{k+1} &\leq -\|g_{k+1}\|^2 + \frac{\|g_{k+1}\|^2}{d_k^T g_k} d_k^T g_k + 2\psi \frac{\|g_{k+1}\|^2}{d_k^T g_k} d_k^T g_k + \frac{\|g_k\|^2}{d_k^T g_k} d_k^T g_k \\
&\quad - \theta_k \sigma \frac{\|g_{k+1}\|^2}{d_k^T g_k} d_k^T g_k - 2\psi \theta_k \frac{\|g_{k+1}\|^2}{d_k^T g_k} d_k^T g_k - \theta_k \sigma \frac{\|g_k\|^2}{d_k^T g_k} d_k^T g_k
\end{align*}
\]

\[
\begin{align*}
&- \theta_k \sigma \frac{\|g_{k+1}\|^2}{g_k^T g_k} d_k^T g_k
\end{align*}
\]
\[ g_k^T d_k \leq -c \| g_k \|^2 \]

\[
d_{k+1}^T g_{k+1} \leq -\| g_{k+1} \|^2 + \sigma \| g_{k+1} \|^2 + 2\sigma \psi \| g_{k+1} \|^2 + \sigma \frac{\| g_k \|^2}{\| g_{k+1} \|^2} \| g_{k+1} \|^2 - \theta_k \sigma \| g_{k+1} \|^2 - 2\sigma \psi \theta_k \| g_{k+1} \|^2 - \theta_k \sigma \| g_{k+1} \|^2 + c\sigma \theta_k \| g_{k+1} \|^2
\]

\[
d_{k+1}^T g_{k+1} \leq -\left[ 1 - \sigma - 2\sigma \psi - \frac{\sigma}{\beta_k^{FR}} + \sigma \theta_k + 2\sigma \psi \theta_k + \frac{\sigma \theta_k}{\beta_k^{FR}} - c\sigma \theta_k \right] \| g_{k+1} \|^2
\]

\[
d_{k+1}^T g_{k+1} \leq - C_1 \| g_{k+1} \|^2 \quad 0 < C_1 < 1
\]

\[
C_1 = \left[ 1 - \sigma - 2\sigma \psi - \frac{\sigma}{\beta_k^{FR}} + \sigma \theta_k + 2\sigma \psi \theta_k + \frac{\sigma \theta_k}{\beta_k^{FR}} - c\sigma \theta_k \right]
\]

4-Global convergence.

**Theorem 4.1.**

Let’s suppose that the assumption \( H_1 \) and \( H_2 \) holds. Consider the algorithm (2),(7),(8) where \( 0 \leq \theta_k \leq 1 \) and \( \alpha_k > 0 \) is obtained by the strong Wolfe line search.

If \( \| s_k \| \) tends to zero and there exists non-negative constant \( \eta_1 \) and \( \eta_2 \) such that \( \| g_k \|^2 \geq \eta_1 \| s_k \|^2 ; \)

\[
\| g_{k+1} \|^2 \leq \eta_2 \| s_k \|
\]

and \( f \) is uniformly convex function, then \( \lim_{k \to \infty} g_k = 0 \)

**Lemma 4.1:**

If the assumptions \( H_1 \) and \( H_2 \) are true, we may examine any conjugating gradient (2) or (3), where \( d_k \) is the descent direction and \( \alpha_k > 0 \) is the result of a strong Wolfe line searching to determine the gradient. If

\[
\sum_{k \geq 1} \frac{1}{\| d_{k+1} \|^2} < \infty \quad \text{then}
\]

\[
\liminf_{k \to \infty} \| g_k \| = 0 \quad [12]
\]

**Proof:**

\[
\beta_k^{HMB} = (1 - \theta_k)\beta_k^{BA} + \theta_k \beta_k^{FR}
\]

\[
\beta_k^{HMB} = (1 - \theta_k) \frac{\eta_k^T y_k}{\eta_k^T y_k} + \theta_k \frac{g_k^T g_{k+1}}{g_k^T g_k}
\]
\[ \beta_k^{HMB} \leq \frac{\|y_k\|^2}{-d_k^T g_k} + \frac{\|g_{k+1}\|^2}{\|g_k\|^2} \]

From \( \|y_k\| \leq L \|S_k\| \)
\[ \leq \frac{L^2 \|S_k\|^2}{c \|g_k\|^2} + \frac{\|g_{k+1}\|^2}{\|g_k\|^2} \]
\[ \beta_k^{HMB} \leq \frac{L^2 \|S_k\|^2}{c \eta_1 \|S_k\|^2} + \frac{\eta_2 \|S_k\|}{\eta_1 \|S_k\|^2} \]
\[ \beta_k^{HMB} \leq \frac{L^2 \|S_k\|}{c \eta_1 \|S_k\|^2} + \frac{\eta_2 \|S_k\|}{\eta_1 \|S_k\|^2} \]

The new direction
\[ d_{k+1} = -g_{k+1} + \beta_k^{HMB} d_k \]
\[ \|d_{k+1}\| = \| -g_{k+1} + \beta_k^{HMB} d_k \| \leq \|g_{k+1}\| + \beta_k^{HMB} \|d_k\| \]
\[ \|d_{k+1}\|^2 = \|g_{k+1}\|^2 + 2 \beta_k^{HMB} \|g_{k+1}\||d_k\| + (\beta_k^{HMB})^2 \|d_k\|^2 \]
\[ \leq \eta_2 \|S_k\| + 2 \left[ \frac{L^2 \|S_k\|}{c \eta_1 \|S_k\|^2} + \frac{\eta_2 \|S_k\|}{\eta_1 \|S_k\|^2} \right] \eta_2 \frac{\|S_k\|^2}{|\alpha_k|} \]
\[ + \left[ \frac{L^2 \|S_k\|}{c \eta_1 \|S_k\|^2} + \frac{\eta_2 \|S_k\|}{\eta_1 \|S_k\|^2} \right]^2 \frac{\|S_k\|^2}{|\alpha_k|^2} \]

From \( \|S_k\| \leq D \)
\[ \leq \eta_2 D + 2 \left[ \frac{L^2 D}{c \eta_1} + \frac{\eta_2 \|S_k\|}{\eta_1 \|S_k\|^2} \right] \eta_2 \frac{1}{|\alpha_k|} + \left[ \frac{L^2 D}{c \eta_1} + \frac{\eta_2 \|S_k\|}{\eta_1 \|S_k\|^2} \right] \eta_2 \frac{1}{|\alpha_k|^2} \]

let \( \varphi = \eta_2 D + 2 \left[ \frac{L^2 D}{c \eta_1} + \frac{\eta_2 \|S_k\|}{\eta_1 \|S_k\|^2} \right] \eta_2 \frac{1}{|\alpha_k|} + \left[ \frac{L^2 D}{c \eta_1} + \frac{\eta_2 \|S_k\|}{\eta_1 \|S_k\|^2} \right] \eta_2 \frac{1}{|\alpha_k|^2} \)
\[ \therefore \|d_{k+1}\|^2 \leq \varphi \]
\[ \sum_{k=1}^{\infty} \frac{1}{\|d_{k+1}\|^2} \geq \sum_{k=1}^{\infty} \frac{1}{\varphi} = \frac{1}{\varphi} \sum_{k=1}^{\infty} 1 = \frac{1}{\varphi} \sum_{k=1}^{\infty} 1 = \infty = \infty \]
\[ \therefore \lim_{k \to \infty} \inf \|g_k\| = 0 \]
5-Numerical

In this section, we’ll discuss the results of our numerical experiments with the hybrid MB algorithm and compare them to the numerical results of the other two algorithms (FR, BA1) under the strong Wolfe line search, which is based on number of iterations (NI) and number of function evaluation (NF), with iterations ending when \( \| g_k \| \leq 10^{-6} \).

In addition, when the number of variables (n=200,900) was high, we used 75 functions of unconstrained optimization problems. All the graphs in this study were created in Fortran.

The results are shown in Table 1.

**TABLE 1.** list numerical result details.

<table>
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<td>NF</td>
<td>NI</td>
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The percentage of improvement is shown in both tables 2-3

Table 2

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Table 3:

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Figure 1: Based on NI at 200

Figure 2: Based on NI 900
Figure 3: Based on NF 200

Figure 4: Based on NF 900
The statistics above show a comparison of the new algorithm MB with both FR and BA1 in terms of NI, NF. Dolan and More [13] is utilized to demonstrate the outcomes of a newly developed hybrid conjugate gradient algorithm. As a result, we can deduce that the hybrid method is effective.

6-Conclusion:

Based on the hybridization of the two algorithms ($\beta_k^{BA_1}$ and $\beta_k^{FR}$), a new approach termed was introduced in this research for hybrid conjugating gradient in unconstrained optimization.

The qualities of sufficient descent and global convergence of the suggested algorithm have been confirmed by some of the assumptions employed, and the proposed method has been explored both theoretically and practically.

References